Distributed word representations: vector comparison

Chris Potts
Stanford Linguistics

CS 244U: Natural language understanding

Running example

	d_x	d_y
A	2	4
B	10	15
C	14	10

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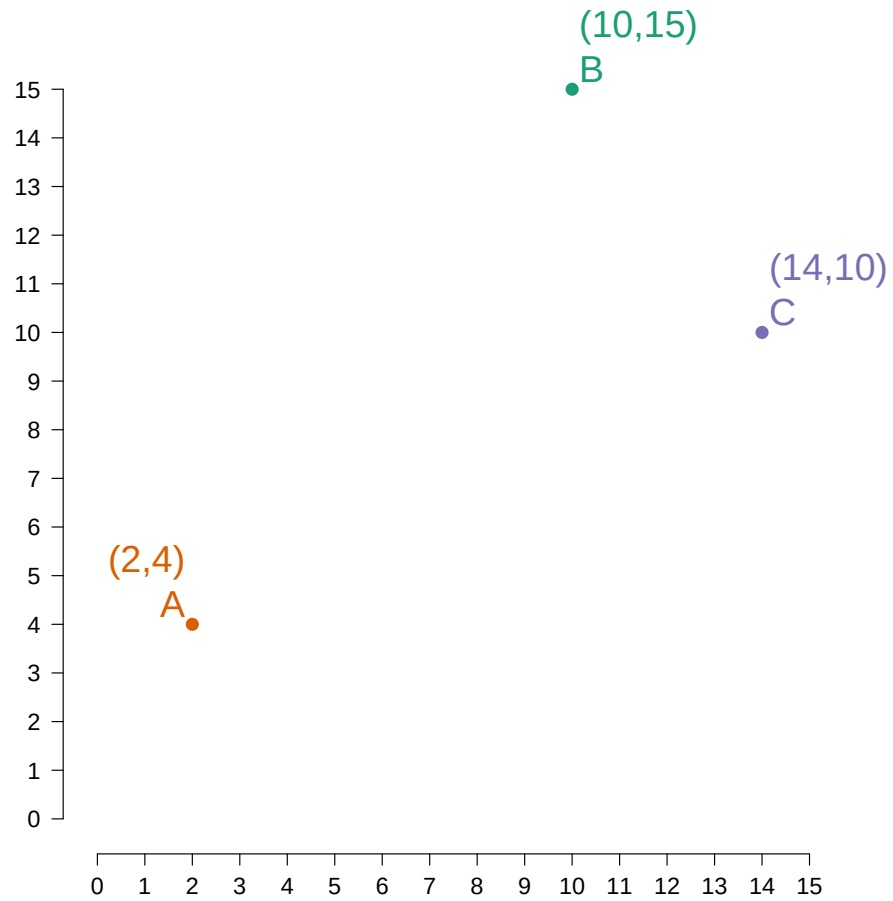

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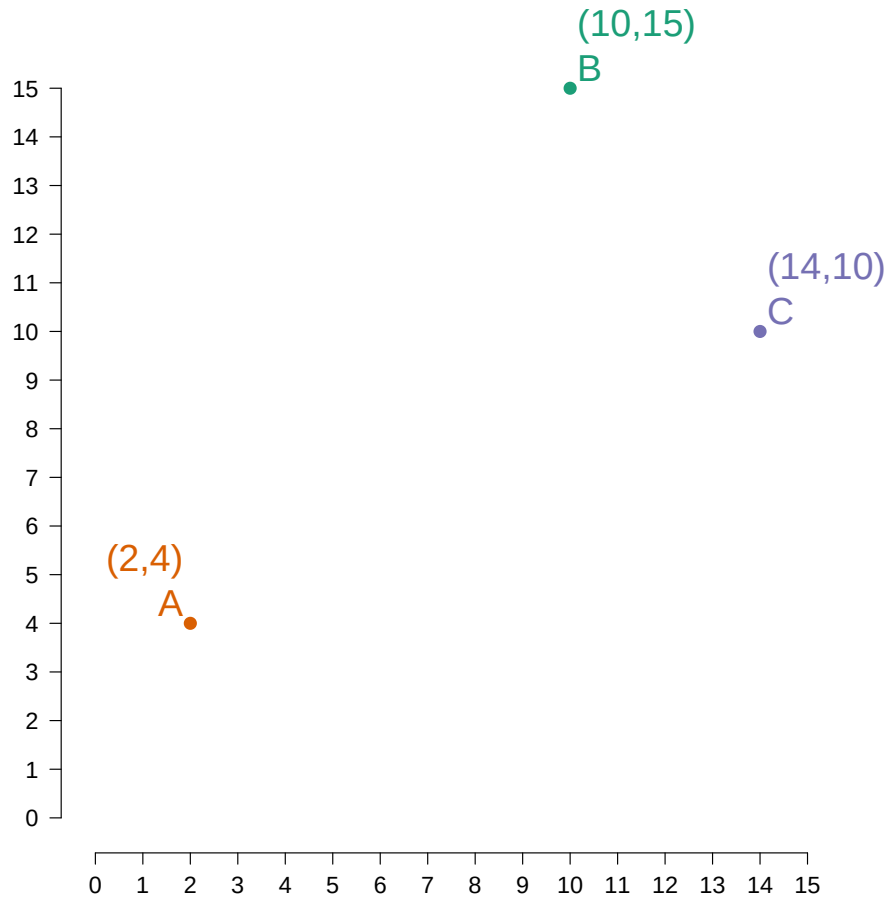

- Focus on distance measures

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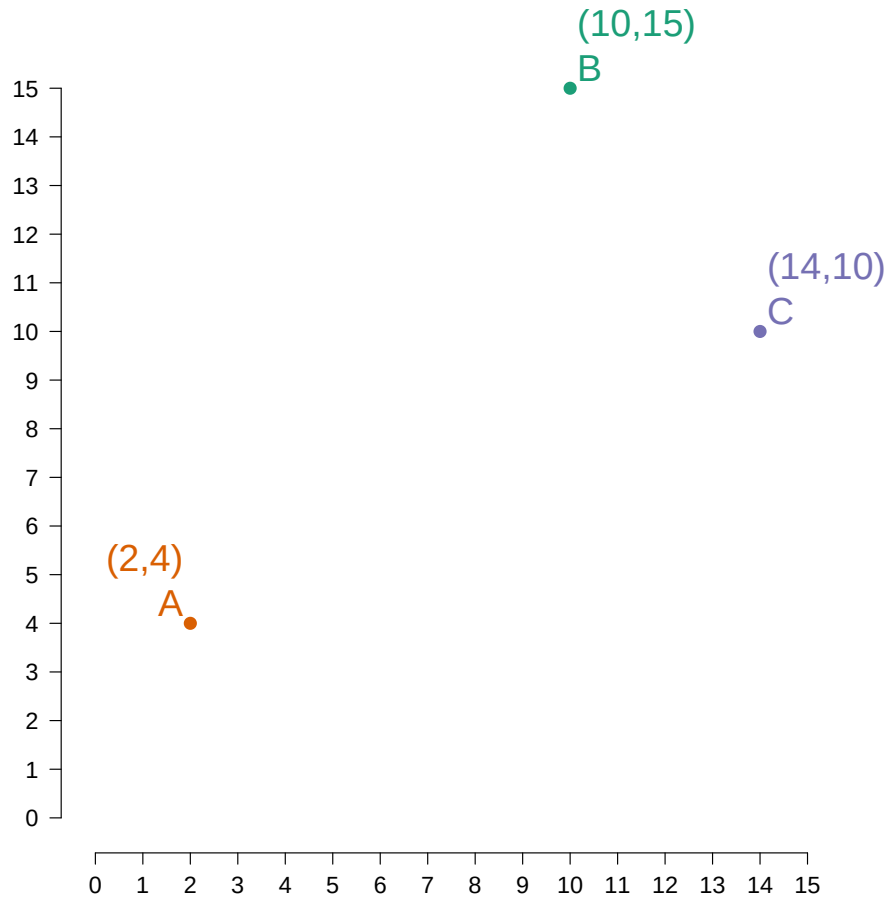

- Focus on distance measures
- Illustrations with row vectors

Euclidean distance

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Between vectors u and v of dimension n:

$$\sqrt{\sum_{i=1}^{n} |u_i - v_i|^2}$$

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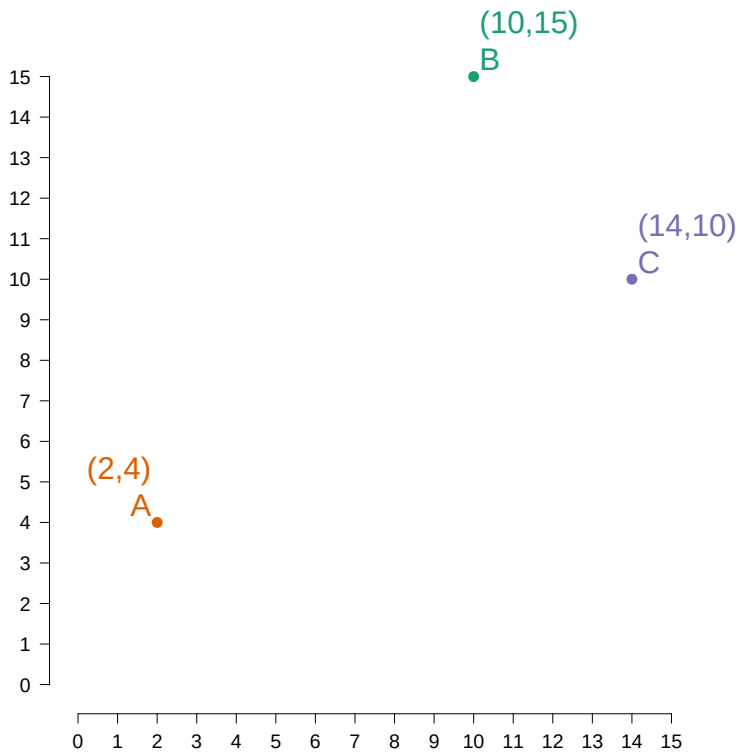

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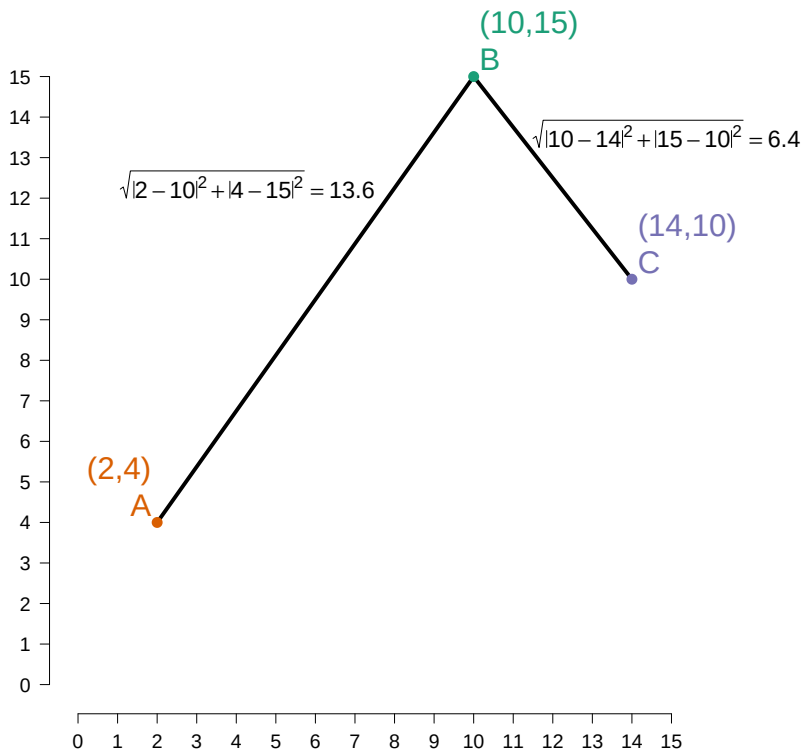

Vector L2 (length) normalization

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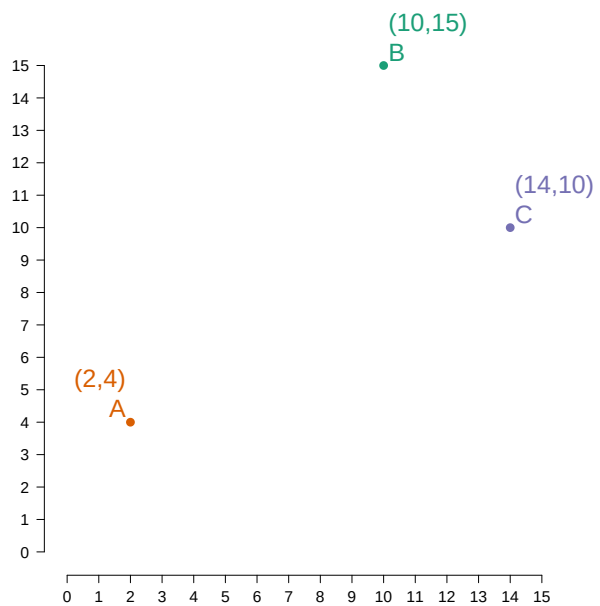

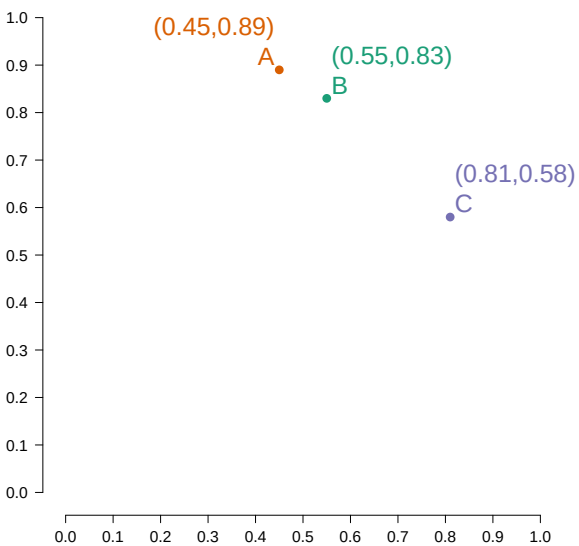

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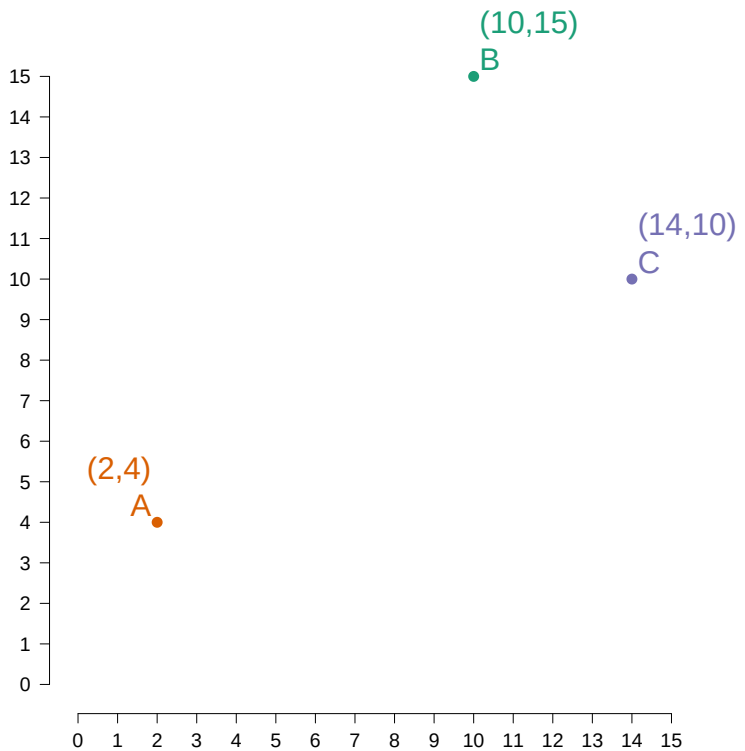

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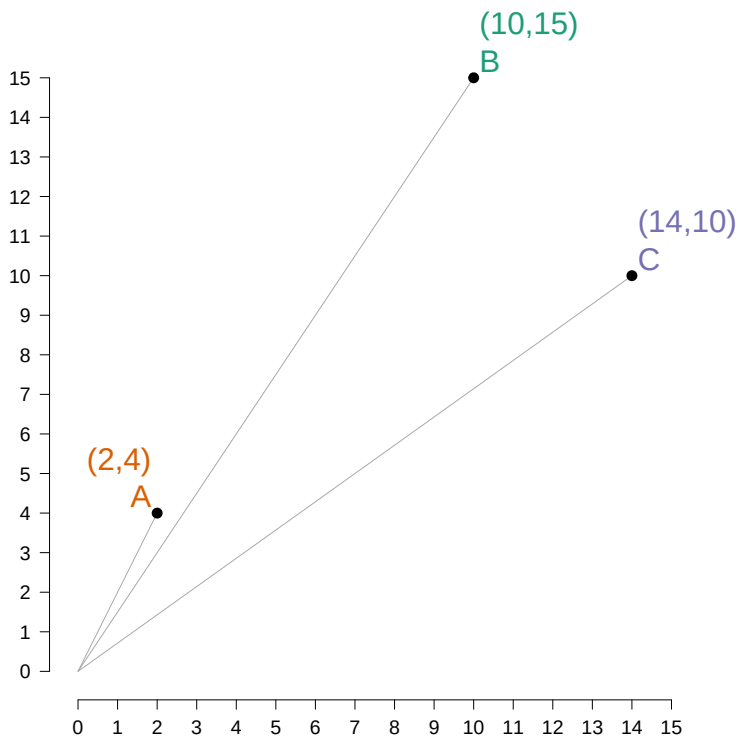

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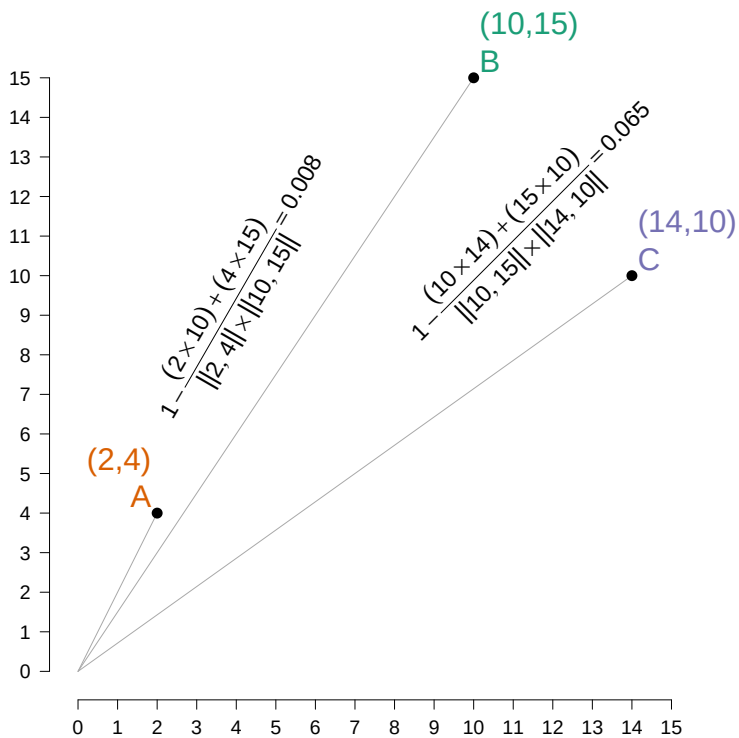

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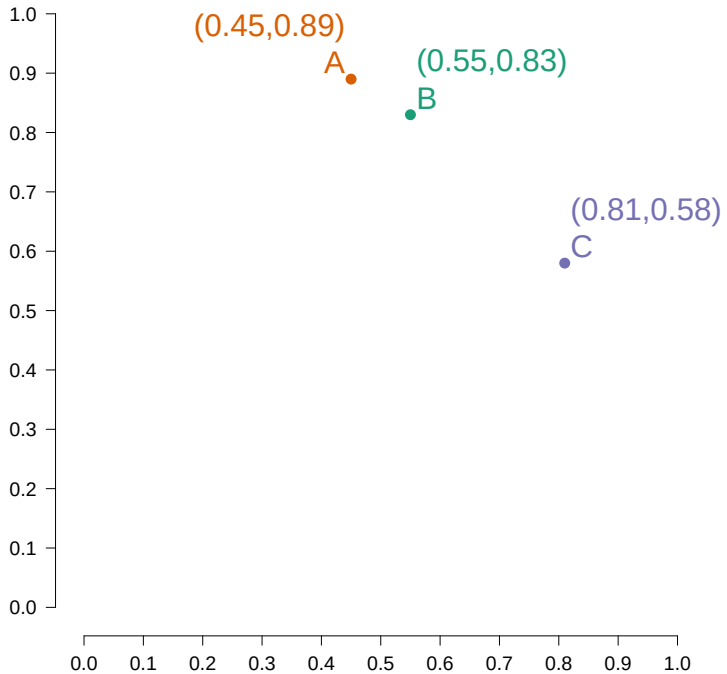

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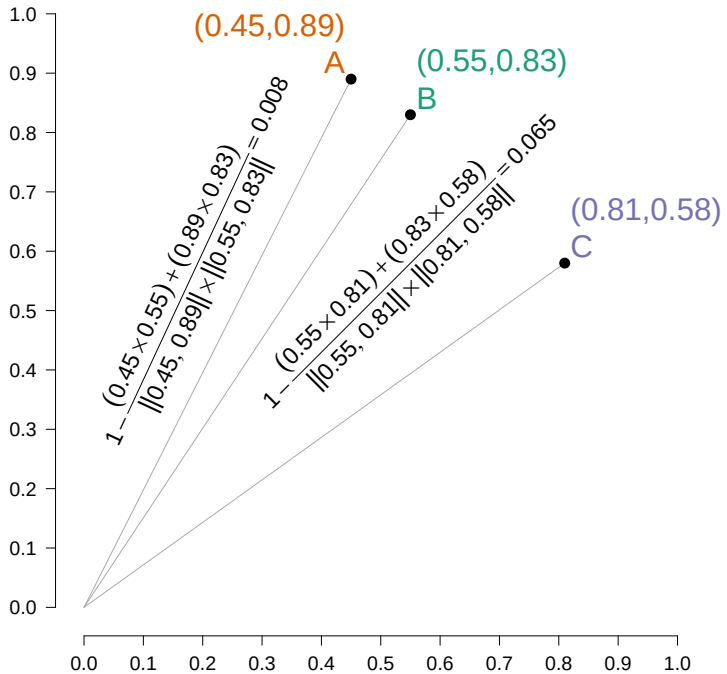

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	A and B closer than B and C?
Euclidean distance	No
Cosine distance	Yes

Other comparison methods

- Manhattan distance
- KL divergence
- Symmetric KL divergence
- KL divergence with skew
- Jensen–Shannon distance
- Matching coefficient
- Dice coefficient
- Jaccard coefficient